Thresholding (Binarization)
Idea

• Gray level $\iff$ Bi-tonal

• Simplest form of segmentation

• Like any segmentation, the key is how you choose the predicate $P(R_i)$ to threshold the intensities
Thresholding

- Segment image into foreground and background
  \[ g(x,y) = \begin{cases} 
  1 & \text{if } f(x,y) \text{ is foreground pixel} \\
  0 & \text{if } f(x,y) \text{ is background pixel} 
\end{cases} \]

- Procedure results in multiple regions, \( R_i \)

  \[ \bigcup_{i=1}^{n} R_i = R \]

  \[ R_i \cap R_j = \emptyset \text{ for all } i \text{ and } j, i \neq j \]

- if \( R_i = 1 \) foreground, else \( R_i \) is a background region
Global Thresholding

• Choose a global threshold, $T$

• Pass in logic operator (P)
  - IS_LESS_THAN, IS_GREATER_THAN

\[ g(x,y) = \begin{cases} 
1 & \text{if } P(f(x,y), T) = \text{TRUE} \\
0 & \text{otherwise} 
\end{cases} \]
Example

Original

Under

Good

Over
Choosing Global Threshold T

- Examine the histogram
- Hopefully we will find 2 dominant peaks
- Set threshold between these peaks
Histogram

Multiple peaks
Where do you set the threshold?
Automatic Threshold Selection

• Why?
  - Automation
    • Human is not always available
  - Repeatability
    • Provides a standard approach to choosing thresholds
      - For automation
    • Others can reproduce your results
      - In research, this is important
      - Often overlooked!
Iterative Method for Finding T

1. Estimate value of T (start with mean)
2. Divide histogram into two regions, R1 and R2 using T
3. Calculate the mean intensity values $\mu_1$ and $\mu_2$ in regions R1 and R2

4. Select a new threshold $T = (\mu_1 + \mu_2)/2$

5. Repeat 2-4 until the mean values $\mu_1$ and $\mu_2$ do not change in successive iterations
Example

- Iteration 0: $T=146.0$ $u_1=90.1$ $u_2=182.0$
- Iteration 1: $T=136.0$ $u_1=85.3$ $u_2=179.9$
- Iteration 2: $T=133.0$ $u_1=83.7$ $u_2=179.2$
- Iteration 3: $T=131.0$ $u_1=83.1$ $u_2=178.8$
- Iteration 4: $T=131.0$ $u_1=83.1$ $u_2=178.8$
Optimal Thresholding

• In optimal thresholding, a criterion function is devised to yield some measure of separation between regions

• Two regions are considered to be the foreground and background
  - (we can call these “classes”)
Otsu Method

- Based on selecting the lowest point between two classes
  - Formulated as discriminant analysis; criterion function is used as a measure of statistical separation

- Analysis of the variance (variance = $\text{std}^2$)
  - Separately compute the variance of the two classes
    - $\sigma^2_T$ = total variance
    - $\sigma^2_W$ = within-class variance
  - The variation of the mean values for each class from the overall intensity mean of all pixels defines a between-classes variance ($\sigma^2_b$)
\[ \sigma_b^2 = P_1 \sigma_1^2 + P_2 \sigma_2^2 \]
Otsu Method

• Step through all intensity levels in histogram
  – Set T (Threshold) to current intensity level
  – Compute between-class variance

• Desired threshold level maximizes between-class variance

Matlab uses this method in a function called:

\[ T = \text{graythresh}(I); \]
Example

Mean
143

Iterative
131

Otsu
130
Problems with Global Histogram

• Large constant image areas dominate

• Instead, use selected pixels to create the histogram
  - Pixels near boundaries
    • Pixels in the neighborhood high gradient magnitudes
Problems with Global Threshold

• Applies a global threshold
  – Can’t handle changing illumination

• Can give poor results for certain types of images

• We may consider a local approach
Local (Adaptive) Thresholding

• Divide image into sub-images

• Threshold sub images

• Use a sliding window
  - If region is constant, consider it against a global threshold (all black or white)
  
  - If sufficient variance, use Otsu/Iterative method in the window

  - Threshold pixel at the center of the window using local $T$
Example

Sonnet for Lena

O dear Lena, your beauty is so vast
It is hard sometimes to describe it fast.
I thought the entire world I would impress
If only your portrait I could compress.

Also! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
And for your lips, sensual and tactile

Thirteen Guests found not the proper fractal
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when filters took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

Thomas Colburn
Threshold Selection Based on Boundary Characteristics

• First order derivative to tell us when we have encountered a boundary
  - Gradient

• Second order describes the transition
  - Background to foreground
  - Foreground to background

  - Use the Laplacian operator

\[ \nabla^2 f = 4z_5 - (z_2 + z_4 + z_6 + z_8) \]
Threshold Selection Based on Boundary Characteristics

- Create a new image

\[ s(x, y) = \begin{cases} 
0 & \text{if } \nabla f < T \\
+ & \text{if } \nabla f \geq T \text{ and } \nabla^2 f \geq 0 \\
- & \text{if } \nabla f \geq T \text{ and } \nabla^2 f < 0 
\end{cases} \]

- where 0, +, - represent three distinct gray levels, and T is a global threshold
Computing the Bi-tonal Image

• Considering a light background
  
• We scan the image $s(x,y)$
  
• A transition from light background to a dark object must be characterized by the occurrence of a “−” followed by a “+”
  - the interior can be “+” or “0”s
  
• A transition from dark object back to the background is
  - “+” followed by a “−”
Computing the Bi-tonal Image

• Thus, we are looking for expression of the nature (. . .)(-,+)(0 or +)(+-)(. . .)

• We set pixels corresponding to (0 or +) to 1
  - 0 otherwise

• If we have light objects on a dark background, reverse the assignment of “+” and “-”

• This algorithm has a global threshold, but it is used based on local information
Example

Original

Threshold via Boundary Characteristics
Summary

• Bi-tonal Thresholding
  - Segments image into foreground and background
  - A "core" image-processing routine
  - Challenge lies in choosing an appropriate threshold

• Global
  - Use histogram to obtain statistics about the image
  - Automatic Techniques
    • Iterative Method
    • Otsu Method, try to find an optimal threshold

• Adaptive
  - Use local information
  - Use boundary characteristics