In this section, we will develop the rules of probability for compound events (more than one event) and will discuss probabilities involving the union of events as well as intersection of two events.
The number of events in the union of A and B is equal to the number in A plus the number in B minus the number of events that are in both A and B.

\[ N(A \cup B) = n(A) + n(B) - n(A \cap B) \]
Addition Rule

- If you divide both sides of the equation by \( n(S) \), the number in the sample space, we can convert the equation to an equation of probabilities:

\[
\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \rightarrow
\]

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]
Addition Rule

• A single card is drawn from a deck of cards. Find the probability that
  • the card is a jack or club.
  • \( P(J \text{ or } C) = p(J) + p(C) - P(J \text{ and } C) \)

\[
\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}
\]
The events King and Queen are mutually exclusive. They cannot occur at the same time. So the probability of a king and queen is zero.

- the card is king or queen

\[ P(K \cup Q) = p(K) + P(Q) - p(K \cap Q) \]

- \[= \frac{4}{52} + \frac{4}{52} - 0= \]
- \[8/52 = 2/13 \]
Mutually exclusive events

If A and B are mutually exclusive then

\[ P(A \cup B) = p(A) + p(B) \]
Use a table to list outcomes of an experiment

- Three coins are tossed. Assume they are fair coins. Give the sample space. Tossing three coins is the same experiment as tossing one coin three times. There are two outcomes on the first toss, two outcomes on the second toss and two outcomes on toss three. Use the multiplication principle to calculate the total number of outcomes: \((2)(2)(2)=8\)

We can list the outcomes using a little “trick” In the far left hand column, write four H’s followed by four T’s. In the middle column, we write 2 H’s, then two T’s, two H’s, then 2 T’s. In the right column, write T,H,T,H,T,H,T,H . Each row of the table consists of a simple event of the sample space. The indicated row, for instance, illustrates the outcome \{heads, heads, tails\} in that order.

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To find the probability of at least two tails, we mark each row (outcome) that contains two tails or three tails and divide the number of marked rows by 8 (number in the sample space). Since there are four outcomes that have at least two tails, the probability is 4/8 or ½.
Two dice are tossed. What is the probability of a sum greater than 8 or doubles?

\[ P(S > 8 \text{ or doubles}) = P(S > 8) + P(\text{doubles}) - P(S > 8 \text{ and doubles}) = \frac{10}{36} + \frac{6}{36} - \frac{2}{36} = \frac{14}{36} = \frac{7}{18}. \]

- \((1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\)
- \((2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\)
- \((3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\)
- \((4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\)
- \((5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\)
- \((6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\)

Circled elements belong to the intersection of the two events.
Complement Rule

Many times it is easier to compute the probability that A won’t occur then the probability of event A.

\[ P(A) + p(\text{not } A) = 1 \rightarrow \]

\[ P(\text{not } A) = 1 - P(A) \]

• Example: What is the probability that when two dice are tossed, the number of points on each die will not be the same?

• This is the same as saying that doubles will not occur. Since the probability of doubles is \( \frac{6}{36} = \frac{1}{6} \), then the probability that doubles will not occur is \( 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6} \).
Odds

- In certain situations, such as the gaming industry, it is customary to speak of the odds in favor of an event E and the odds against E.
- Definition: **Odds in favor of event**
  \[ O = \frac{P(E)}{1 - P(E)} = \frac{P(E)}{P(E')} \quad [P(E) \neq 1] \]

- **Odds against E** = \[ \frac{P(E')}{P(E)} \]

  From adds to probability \( P(E) = \frac{a}{a + b} \)

- **Example:** Find the odds in favor of rolling a seven when two dice are tossed.
- **Solution:** The probability of a sum of seven is 6/36. So
  \[
  \frac{P(E)}{p(E')} = \frac{\frac{6}{36}}{\frac{30}{36}} = \frac{6}{30} = \frac{1}{5}
  \]
**Problem 55**

Let $S$ be the set of all lists of $n$ birth months, $n \leq 12$. Then $n(S) = 12 \cdot 12 \cdot \ldots \cdot 12$ ($n$ times) = $12^n$.

Let $E$ = "at least two people have the same birth month".

Then $E' = "no two people have the same birth month"

$$n(E') = 12 \cdot 11 \cdot 10 \cdot \ldots \cdot [12 - (n - 1)]$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot \ldots \cdot [12 - (n - 1)](12 - n)(12 - (n + 1)) \ldots \cdot 3 \cdot 2 \cdot 1}{(12 - n)(12 - (n + 1)) \ldots \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{12!}{(12 - n)!}$$

Thus, $P(E') = \frac{\frac{12!}{(12 - n)!}}{12^n} = \frac{12!}{12^n(12 - n)!}$ and $P(E) = 1 - \frac{12!}{12^n(12 - n)!}$.